LIMIT ANALYSIS OF THE EXTERNAL WALL OF COLOSSEUM

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SUMMARY

The paper aims to evaluate the safety condition of the Colosseum's structures under the vertical loading condition. The study is based on the limit analysis approach developed for masonry structures and adopts a rigid no-tension constitutive model with no sliding. The paper discusses the results of a preliminary investigation referring to the original configuration of the construction.

1. INTRODUCTION

The complexity of the Colosseum's geometry makes the modeling and the definition of simplified structural schemes quite difficult. Moreover the analysis of historical constructions is generally quite complex because of the difficulties concerning the evaluation of loads and the definition of constitutive laws able to model the behavior of materials such as stones, tuff, concrete and masonry, which are characterized by low or zero tensile strengths and damage properties.

Among the main approaches adopted to analyze the behavior of historical constructions, the limit analysis represents an effective tool for understanding the main aspects of the ultimate behaviour and evaluating the ultimate capacity of the constructions.

By adopting a limit analysis approach the present work discusses the results of a preliminary study concerning the structural safety of the external wall of the Colosseum at vertical loading conditions and with reference to the original configuration of the construction. The study adopts a rigid no-tension constitutive model with no sliding [1-2] and it is based on the geometrical data given by the Soprintendenza Archeologica di Roma and the material data reported in [3].

2. THE CONSTITUTIVE MODEL FOR THE MASONRY

Let us assume a rigid no-tension constitutive model for masonry. This assumption, which was firstly introduced by Heyman [4-5], implies that the masonry body behaves as an assemblage of rigid elements kept together by compression forces and crack at regions characterised by tensile stresses.

The no-tension constitutive assumption is certainly verified for the masonry made of rigid blocks with no mortar. However it can be applied also to ancient masonry made of (tuff) tufa blocks or bricks with mortar joints or to the case of *opus caementicium* considering the fact that the mortar becomes weaker during the time and looses its initial tensile capacity. The no-tension constitutive hypothesis can be adopted also for materials characterised by a finite tensile capacity if we consider the fact that the tensile capacity can be suddenly deteriorated because of some events such as dynamic actions which cause cracking.

The masonry constitutive model can be defined in details on the basis of the hypotheses introduced by Heyman [4-5] in the analysis of the strength of masonry structures and expressed through the following expressions:

$$\underline{?}\ ?\ 0 \qquad (1)$$

$$\underline{?}^{(f)}\ ?\ 0 \qquad (2)$$

$$\underline{?}\ ?\underline{?}^{(f)}\ ?\ 0 \qquad (3)$$

The condition (1), referred to the principal components of the stress tensor $\underline{?}$, defines the domain Y of the admissible stresses and indicates the absence of tensile stresses. As a consequence if P is a point of the masonry body, $\underline{n}$ is the outward normal at the surface where P belongs and $\underline{t}_n$ is the associated stress vector, the equation (1) expresses the convexity condition for the limit surface that is:

$$\underline{t}_n\ ?\underline{n}\ ?\ 0 \qquad (4)$$

The condition (2), referred to the principal components of the cracking strain tensor $\underline{?}^{(f)}$, defines the domain Y' of the admissible strains and indicates the absence of contractions. Finally the normality rule (3) indicates that cracking can develop only at points and along directions where the compression stresses are zero. It expresses the absence of internal dissipation in correspondence of the cracking states.

The convexity condition of the limit surface and the normality rule make it possible the extension of the limit analysis procedure to the masonry structures.

3. ANALYSIS OF THE EXTERNAL WALL

The original configuration of the Colosseum was constituted by eighty columns made of travertine blocks without mortar placed along an approximate elliptical curve with the diameters equal to 188 m and 155 m. The columns were connected by means of arches and architrave at three different levels which supported the four vaults of the ambulatory. At the top of the last level the external wall is characterised by the presence of the *attico* with forty rectangular windows.

The structure shows a high shape resistance which prevents single parts of the wall from being overturned outwards due to the thrusts transmitted by both arches and ambulatory's vaults. This can be explained by the following reasons. First, the single parts of the external wall and the consequent sliding of their interfaces would not rotate because of the high friction resistance caused by the compactness of masonry and the compression transmitted by the arches. Second, each partial mechanism, involving only single parts of the wall, is not compatible with the material constitutive assumptions because it would cause a closure of the stone voussoirs and a consequent penetration of the materials. Thus a compatible mechanism should involve the rotation of the entire wall outwards as it is shown in Figure 1. In this case the wall is subjected to hoop stretches which cause vertical cracking.

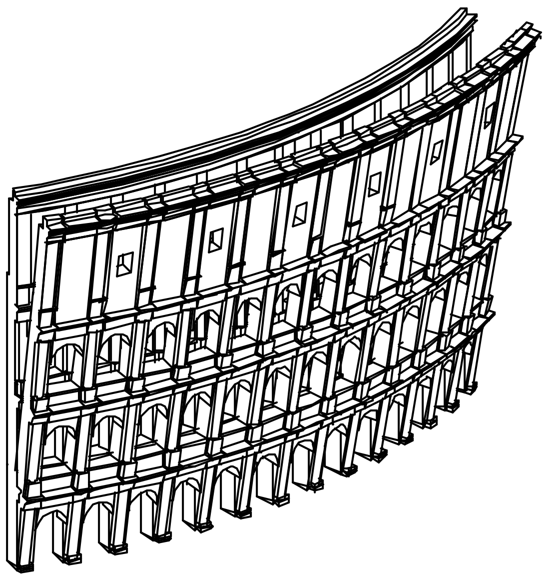
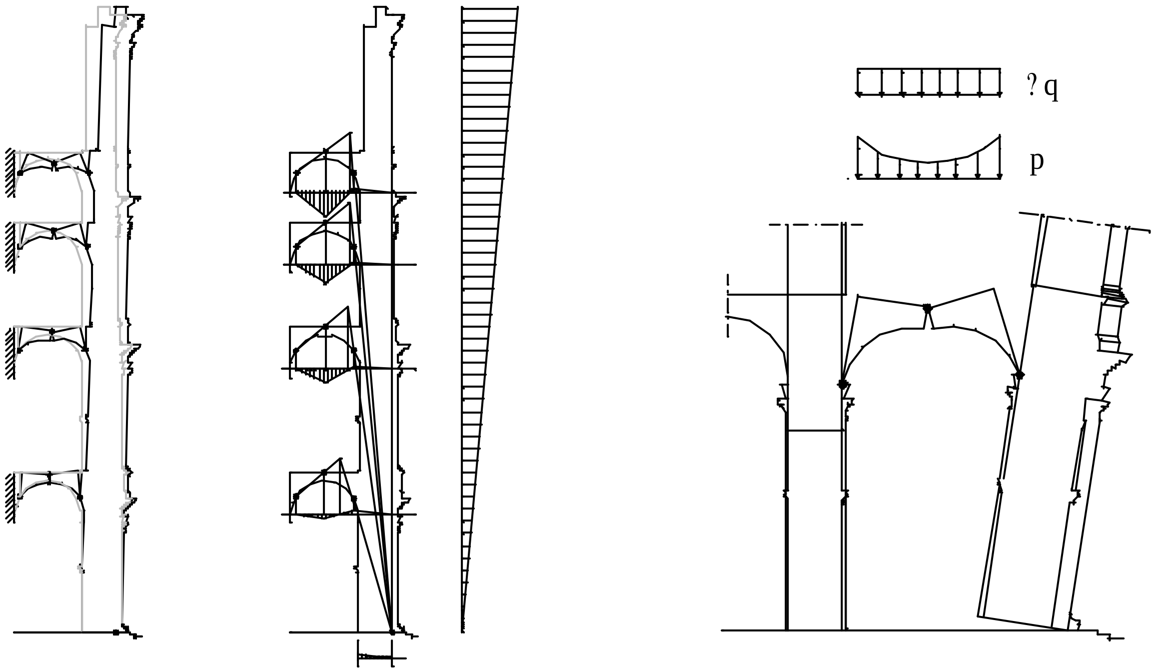


Figure 1: The mechanism of the external wall

Figure 2a represents the mechanism with reference to a radial section of the construction. Figure 2b illustrates the vertical and the horizontal components of the displacement and Figure 2c shows the detail of the ambulatory's vault at the first level subjected to the dead load p and the live load ?q.



a) b) c)

Figure 2: a) radial section of the mechanism; b) vertical and horizontal components of the displacements; c) detail of the ambulatory's vault at the first level.

The evaluation of the collapse load $?_c q$ is obtained by defining the function $?q(x_1,\ldots,x_{12},d)$ being x_i, with i=1..12, the abscissa of the generic hinge formed at the intrados or the extrados of the vaults and d the distance, from the external edge of the wall, of the hinge developed at the bottom which is assumed different from zero to account for the finite compression capacity of the masonry.

By applying the kinematic theorem the function ?q is given by:

$$? \, q \, ? \, \frac{L_{dead}}{L_{live}} \tag{5}$$

The works are evaluated with reference to the structural module shown in Figure 3 and characterised by the radial section reported in Figure 2.

The work L_{dead} is given by:

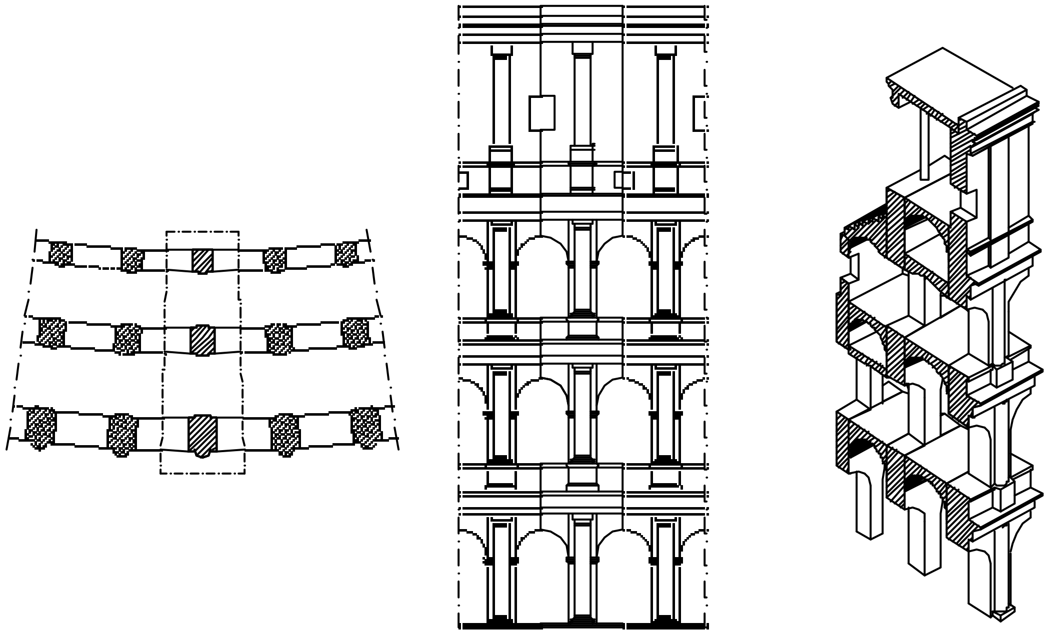
$$L_{dead} \; ? \; L_{p1} \; ? \; L_{p2} \; ? \; L_{p3} \; ? \; L_{p4} \; ? \; L_{Sr(p)} \; ? \; L_W \tag{6}$$

being L_{p1}, L_{p2}, L_{p3}, L_{p4} the works done by the dead loads applied on the four ambulatory's vaults, L_P the work done by the weight P of the wall, $L_{Sr(p)}$ the work done by the radial components $S_r(p)$ of the thrusts $S_c(p)$ transmitted by the arches at dead loads (Figure 4).

The work L_{live} is given by:

$$L_{live} ? L_{q1} ? L_{q2} ? L_{q3} ? L_{q4} ? L_{Sr(q)} \qquad (7)$$

being L_{q1}, L_{q2}, L_{q3}, L_{q4} the works done by the unit live loads applied on the four ambulatory's vaults, $L_{Sr(q)}$ the work done by the radial components $S_r(q)$ of the thrusts $S_c(q)$ transmitted by the arches at unit live loads (Figure 4).

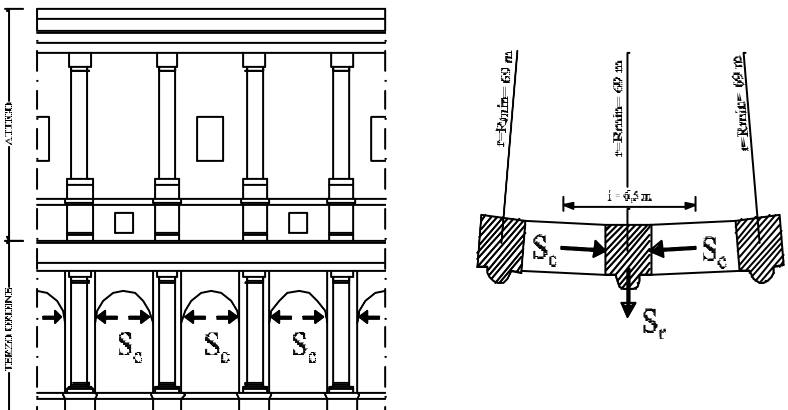


a) b) c)

Figure 3: Structural module of the external wall: a) plan; b) view; c) axonometry

The expression of ?q clearly depends on the position of the hinges defining the mechanism. The minimum of the function ?q(x_1,….,x_{12},d), evaluated with the constraint expressing the equilibrium of the wall in the vertical direction at a fixed level of the masonry compression strength, provides the collapse load $?_c$q related to the set of the analysed mechanisms.
It is worth noting that the previous analysis should be extended to others loading conditions and different mechanisms involving also the inside and the intermediate walls. However, under vertical loading conditions, the inside wall is sustained by the radial walls which prevent the collapse because of their high stiffness.
With reference to the vertical loading condition analysed in this work, the collapse load $?_c$q, given by (5) and related to the set of mechanisms involving the external wall, is slightly greater than 1 t/m^2. It is obvious that if the live load acts only on one vault the collapse load becomes

greater. This highlights a high resistance of the structure under the analysed vertical loading condition. It is worth observing that the previous analyses neglect the soil deformability and the soil structure interaction effects.



$$S_r = \frac{S_c}{r} \cdot i = q_n \cdot i$$

Figure 4: Radial components of the thrusts

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