A PROPOSED METHODOLOGY TO EVALUATE THE SAFETY LEVEL OF VAULTED STRUCTURES IN SEISMIC AREAS: THE CASE STUDY OF S. FRANCIS' IN ASSISI

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SUMMARY

Arches and vaults are with no doubts the most challenging structures, among the historic typologies, from many different point of views, but in particular from the point of view of the engineering assessment of their structural safe.
When the way of construction, the mechanical properties of the materials and a number of structural details are not well known, as it is in the case of historic monuments, the procedure to analyse the structure with different hypotheses and models and to compare the related results is the only way to have reliable engineering assessment on the safety of the structure.
This is the main frame of the work presented in the paper, that is a part of a more comprehensive structural analysis approach still in progress on the behaviour in seismic zone of structures shaped by vaults and in particular gothic vaults.

1. INTRODUCTION

The current confidence in the ability to provide buildings with adequate seismic resistances does not extend back to historic masonry structures. As a matter of fact for minor masonry constructions, like traditional abode or vernacular building, mostly with horizontal timber floors, real case observations and full or scale tests produced in the last twenty years a number of information on their seismic performance, and the knowledge how to proceed for seismic retrofitting procedures [1,2,3,4,5,6,].
When dealing with the seismic response of more complex structures, like churches with slender pillars or vaulted roofs, the current simplified analysis methods are no more pertinent, no experimental tests are available, but only, unfortunately some actual case history of damages and collapses, like the Basilica in Assisi.
The need of better understanding the behaviour under seismic actions of such complex structures is not only urgent, but of primar importance being the only realistic way to prevent future irreparable collapses.

The paper presents a part of an analysis work undergone and numerically implemented on the S. Francis Basilica, by discussing, proposing and implementing different hypotized structural models and behaviours, which take under account either the whole structure and, subsequently significant part of them. In particular arch limit analysis under horizontal action of arches is described in the paper as an extension of that proposed by Heyman. The model and related results are compared and discussed with an alternative two dimensional F.E. incremental non-linear analysis. In the last paragraph the main steps of the procedure to extend each of the two models to the three dimensions are illustrated.

The experience of the authors asserts that this procedure should be the paramount tool to suggest right and adequate structural provisions for the monument, to prevent irreparable failures like those occurred to the webs frescoed by Giotto and Cimabue.

2. STRUCTURAL FEATURES AND DEVICES OF THE GOTHIC VAULTS

The extrados of the masonry historic vaults, generally hidden between the vault section and the upper floor, changed during the centuries, according to the architectural typologies of the vaults. The construction technologies and details had to follow static requirements needed to achieve the safety performance of vaults, whose curvature and shape continuously changed from the Romanic typology to the renaissance and baroque periods. In [8, 9] the static significance of extradoss ribs, buttresses and flanges is underlined by numerical investigations for the so-called "flat masonry vaults", with applications on real examples of the XVIII century. In these case the "extradoss material" has to be considered strictly collaborating with the section of the vault. A complete different structural typology pertains to the gothic ribbed vaults, even if, as for the cloister and flat ceiling vault, they can be classified as thin section vaults, according to the Heyman definition (t/R =5%) [10]. As it is known, the construction technology of the gothic vault is based on the main idea of the structural diagonal ribs with webs resting on them [11], with different construction details for the typology of the rib sections and the web-rib supports (fig. 1). The curvature of the vault follows the curvature of the transverse ogive arches, assuring a safe geometrical shape for the static vertical loads. In this case the rib acting just like a simple support may be safe for the web. Similar considerations may be done on the static significance of the filling material, whose influences for the vault structural performance were present and known to the gothic architects. The reins of the vaults have been often provided with regular courses of bricks (or stones) and mortar (fig. 2) and (fig. 3). It must be recalled, however, that the gothic cathedrals were built in countries with no seismic activity, and the only concern for the presence of horizontal actions were due to the static forces at the vault impost with the pillars. More complex behavior and structural requirements pertain to the same typology of vaults in our country.

The pathology of gothic arch-vault may be regarded as an extension, with peculiar aspects, of the main rules responsible of the crack pictures in old masonry. Even in the case of seismic actions, the formation of the so called “plastic hinge”, as stated by Heyman, seems to be pertinent, as shown in figs. 4 and 5.

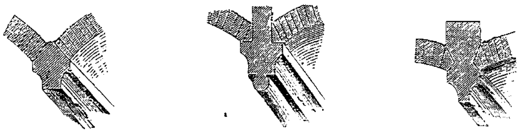


Fig. 1: Different typologies of the rib-web support

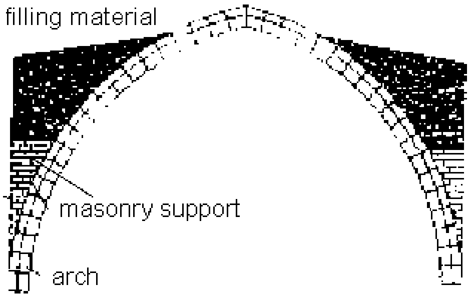


Fig. 2: Geometry of the filling material masonry support vault and arch

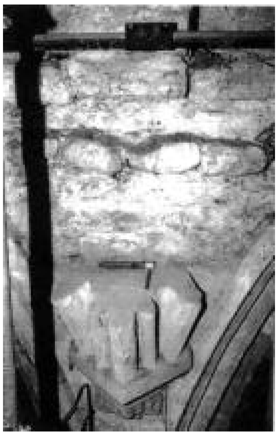
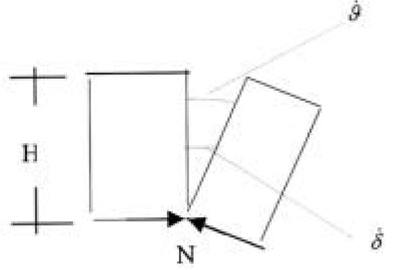


Fig. 3: Regular courses of bricks at the reins

Fig. 4: Extradossal cracks at the key section [12]

Fig. 5: Introdossal crack in the rib section



? ? ? ? H / 2;

M ? ?N ? H / 2;

D ? M ? ? ? N ? ? ? ?N ? (H / 2) ? ? ? N ? ? ? H / 2 ? 0

M ? ?N ? H / 2

M ? ? ? N ? ? ? ? N ? (H / 2) ? ? ? N ? ? ? H / 2 ? 0

Fig. 6 - Model of a masonry arch hinge

3. THE OGIVE ARCH LIMIT ANALYSIS UNDER HORIZONTAL ACTIONS

As it is known, Heyman assumes that the collapse of a masonry arch is the consequence of the formation of a number of hinges sufficient to transform the arch into a kinematic admissible mechanism.

The particular behavior of the hinge, as the opening of two adjacent bricks, is illustrated in the fig. 6 along with the relevant mathematical relationship. Where the moment M has been assumed coherent with the relative rotation velocity ? (a dot stands for time derivative) and the axial force N positive of traction. The first relation links relative rotation of two adjacent sections (corresponding to the adjacent sections of two adjacent bricks) ?, and the centroid opening of the crack ?; the second relation expresses the limit condition in terms of M and N, illustrated in the above figure where, at the limit, the line of the axial force N reaches the bottom fibers of the section; the third relation states that this type of structures have zero dissipation; the fourth and fifth relations relates this simple mechanical model to the general theory of plasticity (normality rule).

We refer to a generic one-degree of freedom (d.o.f.) mechanism, illustrated in the following fig.7. The hinges A, B, C and D delimitate four rigid parts of the arch: part AB rotates around A, supposed fixed; part BC rotates around ? and part CD around D, supposed fixed to the ground.

A simple equation of Principle of Virtual Work permits to compute the load amplifier ? of the

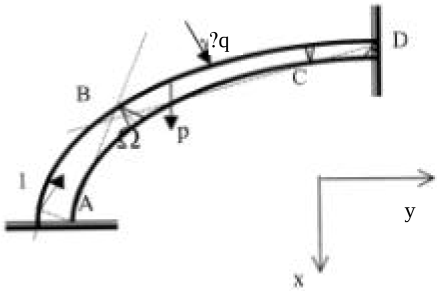


Fig. 7 - One-degree of freedom mechanism

external load q, which ensures the equilibrium between external and internal forces

$$? \int_l (q_x(l)\dot{?}_x(l) ? q_y(l)\dot{?}_y(l))dl ? \int_l p(l)\dot{?}_y(l)dl ? 0$$

where $q_x(l)$ and $q_y(l)$ represent the Cartesian components of the external load at the curvilinear abscissa l; $p(l)$ indicates the self-weight of the arch and the weight carried by the arch at the same abscissa.

In order to estimate the safety factor of the arch against the formation of a kinematic admissible mechanism the following operation are required:

?? Choice of the one d.o.f. mechanism;
?? Discretization of the arch into a set of bricks in order to concentrate external loads (permanent and live) into their centroids transforming the above integral in a suitable summation;
?? Computation of the vertical and horizontal velocities of the bricks centroids due to the mechanism formation.

In the fig. 8 the last point of the entire procedure is shown to be an easy task, such that it can be implemented simply by any standard electronic sheet.

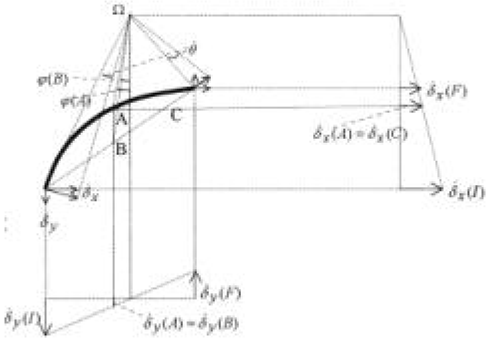


Fig. 8 – Horizontal and vertical velocities along a rigid arch element rotating around ?

It is shown in the following that these velocities vary linearly inside any rigid part of the mechanism.

Therefore it is sufficient to fix the horizontal or vertical velocity of one single hinge in order to determine the entire profile of the displacements of the mechanism.

The simplicity of the procedure, which does not require any sophisticated computer program, together with the basic importance of the information which is able to provide, makes the Heyman approach important and recommendable.

4. THE VAULTS OF THE BASILICA SUPERIORE IN ASSISI

The structural analysis model presented has been applied to the case of the ogive arch of the Basilica Superiore, to find an ultimate structural capability, in terms of either vertical and horizontal (seismic acceleration) loads. The results have been compared to those obtained by an elastic 3D modal analysis of the entire structure and subsequently with results obtained by a for the purpose proposed non-linear incremental model implemented with the Computer Code Struple 2.

From the survey data on the seismic event of 26 Sept. and the modal analysis, which, even in the case of different hypothesis, have stated a value of the fundamental period T= 0.30s., it has been possible, on the base of the spectral response, to determine the acceleration accured on the vaults, as shown in fig. 9.

The limit analysis of the ogive arch under horizontal actions has been performed for different models of the components of the arch, and results are in terms of bounds on ultimate acceleration factor ? :

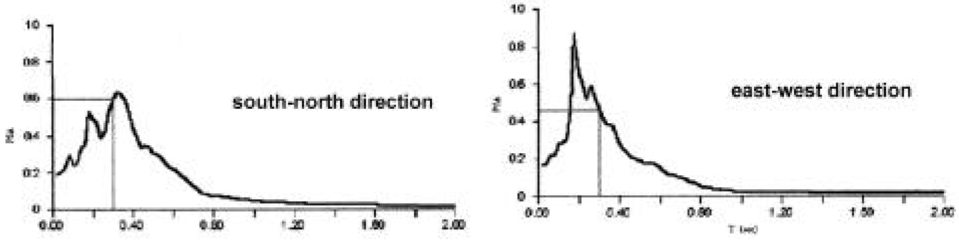


Fig. 9: Acceleration occurred on the Basilica

? is ranging from .3 and .6 considering filling as structural, without or with arch-web collaboration,

? is ranging from .3 and .52 considering no filling, without or with arch-web collaboration. Results in good accordance with those produced by limit analysis, can be read by the incremental ultimate analysis, as briefly shown in fig. 10 and 11.

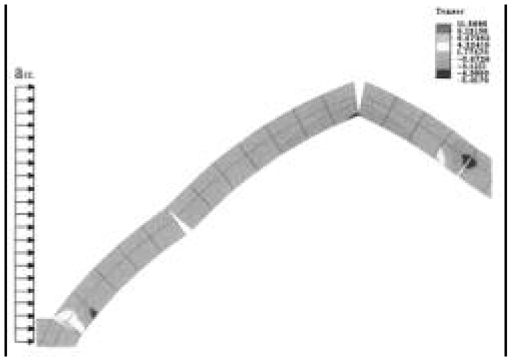


Fig. 10: Arch failure with loose filling material:
? = .304 (no arch-web coll.); ? = .74 (arch-web coll.)

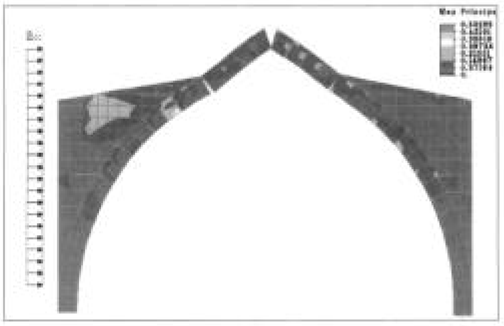


Fig. 11: Arch failure with filling statically active:
? =0.811

As allready mentioned, the work in progress concerns either the extension of the incremental non-linear analysis to the 3D dimensions, by means of DIANA Computer Code, specifically proposed for no-tension materials, and the 3D extension of the limit anslysis as presented here for horizontal loads.

5. CONCLUSIONS

The paper proposes a methodology which, matching together survey data and structural analysis models, is able to give sharp answers to the variety of hypothesis that are asserted on the safety problems of the complex historic monuments.

As shown by the different analytical approaches mentioned in this work, some fundamental structural details are significant to asses the ultimate load bearing capacity of a gothic vault under seismic actions, some other hypothesised as crucial just after the collapse, are of less significant. Stated that, in the particular case of S. Francis Basilica, like as for many other important monuments, the monitoring and survey can give important information on material damages, weak structural portions, cracks evolution, it is possible to use these data and information to adjourn and perform analytical studies on the safety of the structure before and not after its collapse. For the case of the Basilica, in [13], some structural anomalies in the arch of the first nave, pretty important structural weakness and poor conditions of the system arch-web have been already underlined. The study presented takes as parametric coefficients, with the different models described, these kind of weak points, and this global picture can be drawn:

1. the collapse acceleration factor (0.3 ? 0.6 g) given by the limit analysis models is very close to the maximum acceleration expected by the one degree visco-elastic simple model subjected to the ground recorded accelerations of the seismic event of September 26, 1997;
2. the results of the non-linear incremental analysis, give an ultimate capacity in the case of horizontal action, of 0.3 ? 0.7 g in pretty well accordance with the previous analyses;
3. the filling seems to have not a major importance in the collapse;
4. the presence of the r.c. roof had no major influence on the collapse;
5. the load bearing capacity of the gothic vaults is pretty high just respect to the vertical loads;
6. a significant static importance can be attributed to the collaboration vault-arches; it may be assumed that this collaboration, which is due to the bond strength of the mortar and to the friction between the bricks, has deteriorated during time especially because of previous seismic actions.

6. REFERENCES

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